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ON THE INTERPOLATIVE ANOMALIES FOR THE FIRST TEN MINOR PLANETS

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(Ob interpolyatsionnykh anomaliyakh dlya pervykh desyati malykh planet)

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The aim of the present paper is the study of the question of the so-called "interpolative anomaly" for minor planets.

The notion of interpolative anomaly has been introduced by N. D. Moiseyev in his work under reference [1], devoted to the description of a new type of averaged variants of the restricted three body problem. This new type was designated by Moiseyev as the "interpolatively-averaged variant" of the restricted three body problem. It has a well known analogy with the Delaunay-Hill averaged problem, of which it is a substantial generalization.

The main feature of this generalization is the substitution of the so-called Delaunay anomaly by a linear combination of angular expansion variables of a perturbation function selected on the

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* Trudy Gosudarstvennogo Astronomicheskogo Instituta imeni P.K. Shternberga (Transactions of the P.K. Shternberg State Astronomical Institute).

basis of interpolative processing of the observation material, or of the examined minor planet's or comet's osculating elements, corresponding to various moments of some examined time interval. At the same time, this processing is channelled toward searching for a linear combination of the mentioned angles, that would remain "almost constant" over the examined time interval. In the present work we are attempting to establish an interpolative anomaly for the first ten minor planets.

In this context, the present paper may be considered as the first experiment in investigating the possibility of establishing an "interpolative anomaly" without which it is difficult to have an opinion on the practical applicability of the above-mentioned "interpolatively-averaged schemes" of the restricted three body problem, introduced by Moiseyev in the work, to which reference has been made above.

We shall utilize in this work the scheme of the restricted circular problem of three bodies: Sun, Jupiter, Asteroid, where the Sun is the perturbing body, moving along a mean circular orbit with a radius equal to the great semi-axis, and with an inclination and longitude of the orbit plane's node equal to those of the mean elliptical orbit of Jupiter for the year 1850, computed by Hill. The Asteroid is the perturbed body.

#1. UTILIZED OSCULATING ELEMENTS' SYSTEMS OF THE FIRST TEN MINOR PLANETS

The systems of osculating elements for the first ten minor planets, utilized for the computation of interpolative anomalies, are presented below. (See Tables in the next and the following pages).

Table 1

Angular Elements of the Orbits of the First Ten Asteroids *

No.	Oscul.epoch	Eclipt.	M	$\omega^{(e)}$	$\Omega^{(e)}$	$i^{(e)}$
			No. 1. - CERES			
1[6])	1801.0	1801	291° 30′21″.5	64° 58′35″	80° 58′40″	10° 37′56″.6
2[6])	1801.0	1801	290 45 57 .9	65 38 38	80 54 59	10 37 56.0
3[6])	1818 oct. 15.0	1818	240 19 38 .2	67 13 41.9	80 48 32.2	10 38 21.7
4[6])	1850 jan 0.0	1850	161 1 59 .9	67 40 26.9	80 48 5.6	10 37 6.2
5[6])	1854 jan 0	Ep.	113 22 25 .1	68 4 32.6	80 50 50.8	10 37 8.5
6[6])	1854 jan. 0	Ep.	113 18 22 .4	68 5 32.6	80 50 31.1	10 37 4.8
7[6])	1854 jan. 0	1854	113 22 11 .7	68 4 55.5	80 50 31.1	10 37 5.8
8[5])	1913 May. 5.0	d. Ep.	73 53 9 .3	68 40 32.5	80 45 39.4	10 36 55.9
9[6])	1921 Jan. 3.5	1925e	310 20 25 .1	131 12 12.6	23 26 59.3	27 8 6.3
10[6])	1925 Dec. 6.0	1925e	336 41 4 .0	129 51 16.1	23 26 30.2	27 8 53.6
11[5])	1938 Nov. 25.0	1950	270.032	71.245	80.824	10.598
12[5])	1939 Dec. 31.0	1950	356.080	71.123	80.818	10.596
13[5])	1940 March 12.0	1950	11.511	71.112	80.815	10.596
14[4])	1948 Aug. 7.0	1950	310.382	69.863	80.726	10.595
			No 2. PALLAS			
1[6])	1802 31.0	Ep.	41° 16′ 24″.8	309° 12′11″	172° 26′31″	34° 36′59″
2[6])	1803	1803	100 4 41 .0	308 56 5	172 28 8	34 38 20
3[6])	1803	1803	100 26 45 .1	308 39 56.1	172 28 12.4	34 37 28.4
4[6])	1803 5.0	Ep.	133 48 44 .0	308 39 33.3	172 28 20.9	34 37 44.5
5[6])	1810	1810	286 14 59 .8	308 36 21.0	172 31 44.4	34 36 9.5
6[6])	1810 0.0	Ep.	287 51 13 .6	308 40 30.3	172 33 37.4	34 37 42.7
7[6])	1826 24.5	Ep.	133 23 26 .0	308 20 58.8	172 37 9.7	34 35 55.5
8[6])	1830 0.0	Ep.	48 5 46 .8	308 21 12.6	172 39 5.4	34 35 46.5
9[6])	1869 25.0	Ep.	267 46 38 .3	308 58 38.4	172 46 8.3	34 42 18.6
10[6])	1913 5.0	Ep.	71 39 31 .7	309 0 47.9	172 56 47.8	34 42 2.5
11[5])	1916 1.0	1925	261.594**	309.013	173.109	34.699
12[6])	1918 19.0	1918	110 11 16 .7	309 38 51.3	172 53 45.2	34 44 11.3
13[6])	1922 5.0	1925	58 42 13 .5	309 48 31.6	172 58 10.9	34 44 32.6

* The references ahead of the second column bear on the literature sources from which they are taken

** M is given on 1.0.1. 1925.

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No.	Oscul.Epoch	Eclip.	M	$\omega^{(e)}$	$\Omega^{(e)}$	$i^{(e)}$
			№ 2. PALLAS (continuation).			
14[5])	Jan. 1925 янв. 0.5	1925	259.753*	309.695	172.962	34.736
15[6])	Oct 1925 окт. 27.0	1925e	323 49 13.7	323 24 37.5	159 56 5.6	11 46 33.9
16[5])	Sept 1932 сент. 20.0	1925	260.419*	309.281	172.897	34.736
17[5])	Aug. 1938 авг. 31.0	1950	245.220	309.850	173.069	34.804
18[5])	Dec 1939 дек. 31.0	1950	348.968	310.142	173.071	34.818
19[5])	Oct. 1943 окт. 15.0	1950	284.962	310.275	173.039	34.825
20[4])	Aug. 1948 авг. 7.0	1950	301.274	310.007	173.033	34.818
			№ 3. - JUNO			
1[6])	1805.0	1805	349° 18′ 3″.5	242° 14′ 32″.0	171° 4′ 28″.2	13° 4′ 26″.2
2[6])	1810.0	1810	42 23 10 .2	241 59 58.0	171 6 45 .0	13 4 12 .9
3[6])	1810.0	1810	42 26 33 .9	241 52 7.4	171 6 28 .5	13 4 19 .0
4[6])	1811.0	1811	124 33 10 .9	242 5 53.4	171 9 16 .7	13 4 17 .2
5[6])	Dec 1815 дек. 31.0	Ep.	176 58 40 .4	242 4 54.9	171 9 58 .9	13 4 0 .1
6[6])	1819.0	1819	64 12 6 .8	242 26 5.9	171 6 50 .2	13 3 37 .3
7[6])	May 1820 май 11.0	Ep.	176 38 15 .5	242 22 55.4	171 8 11 .1	13 3 47 .2
8[6])	Nov 1826 нояб. 1.0	1810	351 43 27 .3	242 14 16.0	170 56 57 .4	13 3 28 .4
9[6])	Nov 1861 нояб. 21.0	Ep.	4 24 57 .7	243 9 13.6	170 59 49 .7	13 2 58 .8
10[6])	Nov 1861 нояб. 21.0	Ep.	4 24 58 .0	243 [illegible] 18.1	170 59 45 .7	13 2 58 .4
11[6])	Jan. 1900 янв. 0.0	1900	268.60	244.9	170.7	13.03
12[5])	Sept 1913 сент. 10.0	d. Ep.	317. 57 25 .6	245 42 48.0	170 30 12 .7	12 59 52 .8
13[5])	Oct 1916 окт. 1.0	1925	171.90*	244.90	171.05	13.03
14[6])	Oct 1922 окт. 2.5	1922e	346. 26 37 .2	45. 14 22.5	11 20 20 .2	10 49 21 .9
15[5])	Aug. 1926 авг. 3.0	1925	172.261*	245.603	170.430	13.013
16[5])	Oct 1931 окт. 1.0	1925	172.275*	245.609	170.427	13.010
17[5])	AUG 1934 авг. 1.0	1925	170.981*	245.831	170.374	13.020
18[5])	MAY 1938 май 28.0	1950	197.958	245.630	170.721	13.018
19[5])	AUG. 1939 авг. 21.0	1950	299.895	245.486	170.719	13.020

Table 1 (cont. 3)

No.	Oscul. Epoch	Eclip.	M	$\omega^{(e)}$	$\Omega^{(e)}$	$i^{(e)}$
	№ 3. Юнона (продолжение) EUNO - cont'n. -					
20[5])	FEB 1941 февр. 15.0	1950	62°.905	245°.378	170°.711	13°.017
21[5])	MAY 1942 май 9.0	1950	164 .041	245 .477	170 .702	13 .013
22[4])	AUG 1948 авг. 7.0	1950	348 .554	246 .323	170 .496	12 .993
	№ 4 VESTA.					
1[6])	JAN. 1810 янв. 0	1810	216° 4′48″.7	146° 40′ 6″.4	103° 8′20″.5	7° 8′11″.6
2[6])	JAN 1810 янв. 0	1810	216 42 55 .8	146 8 6 .6	103 11 22 .1	7 8 5 .0
3[6])	JAN 1816 янв. 0.0	Ep.	91 25 8 .0	146 29 6 .3	103 13 29 .0	7 8 9 .8
4[6])	1816.0	1818	290 6 46	146 20 6	103 11 38	7 8 9
5[6])	JAN. 1857 янв. 1.0	Ep.	198 20 17 .2	147 10 58 .7	103 23 19 .1	7 8 6 .6
6[6])	JAN. 1857 янв. 1.0	Ep.	198 20 32 .8	147 10 40 .2	103 23 20 .1	7 8 6 .2
7[5])	JAN 1857 янв. 1.0	1950	198.208	147.270	104.596	7.131
8[4])	JAN 1857 янв. 2.0	1950	196.339	148.275	104.007	7.135
9[6])	MAY 1869 май 13.0	Ep.	345 13 50 .4	146 38 57 .2	103 26 23 .3	7 7 53 .4
10[6])	JAN. 1925 янв. 20.0	1925e	107 32 28 .7	235 12 15 .8	18 9 53 .2	22 46 39 .4
	№ 5. ASTREA.					
1[6])	JAN. 1846 янв. 0.0	Ep.	318°40′43″.7	353°58′ 5″.7	141°26′16″.6	5°19′17″.8
2[6])	JAN. 1846 янв. 0.0	1846	318 51 49 .0	353 52 34 .3	141 25 4 .7	5 19 25 .3
3[6])	JAN 1846 янв. 0.0	1846	318 45 3 .3	353 55 32 .4	141 25 14 .6	5 19 22 .7
4[6])	JAN 1846 янв. 0.0	1846	318 45 2 .7	353 55 44 .9	141 25 13 .6	5 19 23 .2
5[6])	JAN. 1850 янв. 0.0	Ep.	306 19 34 .1	353 11 14 .8	141 25 8 .9	5 19 33 .8
6[6])	JAN. 1850 янв. 0.0	1850	306 20 27 .0	353 10 47 .2	141 24 48 .5	5 19 35 .2
7[6])	FEB. 1863 февр. 8.0	Ep.	7 0 52 .5	353 59 42 .3	141 28 2 .7	5 19 6 .6
8[5])	SEP. 1898 сент. 11.0	1910	224 4 1 .2	353 28 9 .3	141 39 24 .5	5 20 3 .2
9[6])	JAN. 1900 янв. 0.0	1900	337 58 [illegible]	354 [illegible]	141 [illegible]	[illegible]
1[6])	JULY 1847 июль 10.0	1847	275° 13′30″.0	236° 18′51″.4	138° 30′ 6″.4	14° 47′ 1″.8
2[6])	JULY 1847 июль 11.0	1847	275 28 41 .7	236 19 25 .2	138 30 5 .3	14 47 2 .2
3[6])	NOV. 1870 нояб. 7.0	1870	338 4 1 .1	236 42 44 .9	138 39 55 .6	14 46 47 .1
4[6])	JULY 1900 июль 3.0	1900	284 20 20 .1	236 56 20 .0	138 39 42 .4	14 48 3 .2
5[5])	DEC 1931 дек. 26.0	1950	43.973	237.383	139.346	14.795
6[5])	FEB. 1940 февр. 21.0	1950	101.095	238.025	138.908	14.755

Table 1 (cont.4)

No.	Oscul. Epoch	Eclip.	M	$\omega^{(e)}$	$\Omega^{(e)}$	$i^{(e)}$
			№ 6. HEBE (cont'n).			
7[5])	MAY 1941 май 6.0	1950	215°.829	238°.113	138°.911	14°.752
8[5])	OCT. 1942 окт. 8.0	1950	351.352	238.189	138.883	14.753
9[4])	AUG 1948 авг. 7.0	1950	187.138	238.192	138.858	14.756
			№ 7. - IRIS. -			
1[6])	JAN 1848 янв. 1.0	Ep.	330° 41′ 54″.0	141° 53′ 3″.3	259° 48′ 10″.2	5° 28′ 15″.9
2[6])	JAN 1848 янв. 1.0	Ep.	330 56 26 .6	141 40 16 .8	259 45 53 .1	5 28 17 .1
3[6])	JAN 1850 янв. 0.0	1850	166 7 39 .3	141 35 18 .8	259 47 43 .6	5 28 2 .3
4[6])	JAN 1850 янв. 0.0	1850	166 7 9 .0	141 35 25 .3	259 47 55 .8	5 28 3 .0
5[6])	JAN 1900 янв. 0.0	1900	9 5 20 .1	141 31 26 .9	260 33 44 .3	5 28 1 .2
6[6])	JAN 1916 окт. 1.0	1925	290.717*	141.490	260.945	5.467
7[6])	OCT 1931 окт. 1.0	1925	289.984*	143.238	259.827	5.485
8[5])	SEP. 1933 сент. 15.0	1925	290.288*	143.363	259.792	5.485
9[5])	SEP 1934 сент. 15.0	1925	289.620*	143.798	259.674	5.497
10[5])	OCT 1935 окт. 1.0	1925	289.016*	143.906	259.676	5.498
11[5])	OCT. 1936 окт. 1.0	1925	289.101*	143.902	259.676	5.498
12[5])	JULY 1938 июль 4.0	1950	167.926	143.819	260.049	5.498
13[5])	JUNE 1939 июнь 30.0	1950	264.537	143.814	260.045	5.498
14[5])	JUNE 1940 июнь 24.0	1950	0.941	143.715	260.016	5.499
15[5])	JUNE 1941 июнь 19.0	1950	97.263	143.702	260.012	5.499
			№ 8. - FLORA			
1[6])	JAN. 1848 янв. 1.0	1848	35°48′ 7″.0	282°42′28″.8	110°18′12″.0	5°53′4″.8
2[6])	JAN. 1848 янв. 1.0	1848	35 47 35 .1	282 43 3 .2	110 18 12 .8	5 53 5.4
3[6])	JAN. 1848 янв. 1.0	1848	35 48 24 .3	282 42 19 .4	110 18 3 .8	5 53 6.2

Table 1 (cont.5)

No.	Oscul. Epoch	Eclip.	M	$\omega^{(e)}$	$\Omega^{(e)}$	$i^{(e)}$
	No 8. - FLORA (cont'n).					
4[6])	JAN 1848 янв. 1.0	1848	35° 54′3″.6	282° 36′39″.7	110° 17′48″.6	5° 53′8″.0
5[6])	JAN 1848 янв. 1.0	Ep.	35 52 49.3	282 38 15.6	110 17 16.7	5 53 7.3
6[5])	JAN 1900 янв. 0.0	1950	5. 68	283. 74	111. 11	5.895
7[5])	OCT 1916 окт. 1.0	1925	242.457*	282.725	111.276	5.881
8[5])	SEP 1934 сент. 15.0	1925	241. 06*	283. 71	110. 79	5.896
	No 9. - METIS					
1[6])	MAY 1848 май 0.0	Ep.	144° 19′41″.2	2° 33′24″.3	68° 27′46″.8	5° 35′48″.6
2[6])	MAY 1848 май 3.0	Ep.	145 6 50 .7	2 33 42 .3	68 27 46 .7	5 35 48 .5
3[6])	MAY 1848 май 3.0	Ep.	145 6 49 .5	2 33 39 .3	68 27 50 .1	5 35 48 .5
4[6])	JUNE 1858 июнь 30.0	Ep.	56 56 39 .9	2 42 15 .0	68 29 30 .5	5 35 57 .8
5[6])	JUNE 1858 июнь 30.0	Ep.	57 4 34 .7	2 22 16 .9	68 31 35 .2	5 35 0 .3
6[5])	JULY 1858 июль 1.0	1950	57.218	2.656	69.686	5.603
	No 10. - HYGIEA					
1[6])	SEP 1851 сент. 17.0	1850	121° 35′.46	303° 53′.59	287° 8′.56	3° 47′.76
2[6])	SEP 1851 сент. 17.0	1850	121.5316	303 6246	287 4578	3 7917
3[5])	DEC 1898 дек. 20.0	1910	291° 20′17″.9	308° 57′0″.0	285° 58′13″.6	3° 48′51″.6
4[5])	OCT 1916 окт. 1.0	1925	179. 367	308. 923	286. 207	3. 815
5[6])	JUNE 1921 июнь 9.5	1925e	311 40 39 .7	238 26 6 .0	351 12 40.0	24 44 26 .3
6[5])	JUNE 1921 июнь 10.0	1925	181. 890*	304. 661	285. 748	3 810
7[5])	JAN. 1925 янв. 0.5	1925	181. 962*	304. 885	285. 688	3. 809
8[5])	NOV. 1931 нояб. 5.0	1925	184. 055	304. 376	285. 568	3. 806
9[5])	FEB 1938 февр. 1.0	1950	306. 697	305. 540	285. 806	3. 800
10[5])	SEP 1940 сент. 4.0	1950	113. 586	306. 022	285. 770	3. 799
11[4])	AUG 1948 авг. 7.0	1950	259. 084	310. 265	285. 384	3. 813

Here the first column indicates the system number of the osculating elements for every asteroid; the second column shows the osculation epoch. The third column gives the epoch of the equinox, to whose ecliptic the elements ω, Ω and i are related. The sign „e", whenever it stands near that epoch, indicates that the corresponding elements are related to the equator. Further we have: M, $\omega^{(e)}$, $\Omega^{(e)}$ and $i^{(e)}$. The upper index „e" at the three latter indicates that they are either related to the equator or to the ecliptic. But what is lacking in the preceding Table, it is those osculating element systems that are clearly erroneous as a result of computations, though given in literature, of ω, Ω and i that have been related to ecliptics of different equinoxes.

2. UTILIZED ELEMENT SYSTEM OF THE MEAN ELLIPTICAL ORBIT OF JUPITER

This system of elements, computed by Hill for the year 1850, is presented below:

$$\left.\begin{array}{l} \lambda = 159^\circ 56' 25''.05, \\ \pi = \ \ 11\ 54\ 26\ .72, \\ \Omega = \ \ 98\ 55\ 58\ .16, \\ i = \ \ \ 1\ 18\ 41\ .81. \end{array}\right\} 1850.0 \qquad \begin{array}{r} e = 0.04825382, \\ n = 299''.12837656, \\ \lg a = 0.7162373716. \end{array}$$

The system of elements of Jupiter's orbit was borrowed by us from the "Astronomical Yearbook". The elements ω, Ω, and i were calculated over again relative to the ecliptic of the year 1950.0 equinox to be used as a basis for further computations:

$$\left.\begin{array}{l} \Omega = 99^\circ 46' 42''.6 \\ \omega = 273\ 31\ 28\ .0 \\ i = \ \ 1\ 18\ 29\ .2 \end{array}\right\} 1950.0$$

#3. ON THE COMPUTATION OF AUXILIARY QUANTITIES

In order to interpolate the values of the anomalies, it is necessary to know those of the mean anomaly for the various moments of time; of the angular distance of the perihelion from the node, specifically — from the node of the plane of the asteroid's orbit in the orbital plane of Jupiter, and not in the plane of the ecliptic; of the node's longitude computed from the Jupiter's radius-vector in the latter's orbital plane to the line of nodes of the orbital plane of the asteroid in the orbital plane of Jupiter. Since all the three quantities are not available in the Table brought out in # 1, we must proceed with their computation.

The first quantity — the mean anomaly of the asteroid M — is obtained by accounting for the number of revolutions made by the minor planet from the moment of osculation for the first system of its elements, and by corresponding additions to the values M of multiples of 360°.

As to the remaining two quantities — the angular distance of the perihelion from the node ω and the longitude of the node Ω — the composition of a special computing scheme for them was made necessary, for these quantities ω, Ω are given in the table of elements of minor planets relative to the ecliptic plane, and not to that of Jupiter's orbit, and for the ecliptic of various equinoxes. Thus emerges the problem of translating these elements from the ecliptic planes of different epochs to the plane of Jupiter's mean circular orbit.

This problem is solved in two stages: first of all one must compute these elements relative to the ecliptic of any one epoch, for example as is now the case, relative to the ecliptic of the year 1950, then transfer these elements from the ecliptic plane of the year 1950 to the plane of Jupiter's mean circular orbit. In conformity with the computing schemes, such an operation, as we shall see in a moment, must not only be conducted with elements ω and Ω, but also with the orbit inclination $\underline{i}$ of the asteroid.

The first half of the problem is solved with the aid of the following formulae (see for instance, the book by A. Ya. Orlova and B. A. Orlova [2]) :

$$\left.\begin{aligned} \Omega_{1950}^{(e)} &= \Omega_t^{(e)} + a - b\sin\left(\Omega_t^{(e)} + c\right)\operatorname{ctg} i_t^{(e)}, \\ i_{1950}^{(e)} &= i_t^{(e)} + b\cos\left(\Omega_t^{(e)} + c\right), \\ \omega_{1950}^{(e)} &= \omega_t^{(e)} + b\sin\left(\Omega_t^{(e)} + c\right), \end{aligned}\right\} \qquad (1)$$

where the values of the coefficients $\underline{a}$ and $\underline{b}$, and also of the quantity $\underline{c}$, are given for different equinoxes in Table VII, at the end of the book referred-to [2].

With the help of the obtained values of $\omega_{1950}^{(e)}$, $\Omega_{1950}^{(e)}$ и $i_{1950}^{(e)}$ one may also resolve the second part of the problem. We shall examine for that purpose the spherical triangle formed on the celestial sphere by the lines of intersection with it of the ecliptic planes, of Jupiter's mean circular orbit and of the osculating orbit of the asteroid. (Fig. 8).

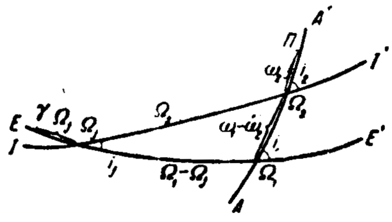


Fig. 8

EE' — Position of the ecliptic in the year 1950; Ω — point of the spring equinox; II' — position of Jupiter's mean orbital plane; Ω_j — Node of Jupiter's mean orbit; i_j — its inclination; AA' — Position of the osculating plane of the asteroid; π — Perihelion of the orbit of the asteroid; Ω_1 — asteroid orbit's node situated in the ecliptic; Ω_2 — asteroid orbit's node situated in the mean orbital plane of Jupiter; Ω_2, i_2, ω_2 — elements characterizing the position relative to Jupiter's mean circular orbit of the osculating plane and of the ellipse of asteroid's orbit; * — elements characterizing the position relative to the ecliptic of the osculating plane, and the ellipse of the orbit of the asteroid. * Ω_1 i_1 and ω_1 .

From the spherical triangle $\Omega_1, \Omega_2, \Omega_j$ one may determine the quantities Ω_2, $\omega_1 - \omega_2$ and i_2 by means of the following spherical trigonometry formulae:

$$\left.\begin{aligned} \operatorname{tg} \Omega_2 &= \frac{\sin(\Omega_1 - \Omega_j)}{-\operatorname{ctg} i_1 \sin i_j + \cos i_j \cos(\Omega_1 - \Omega_j)}, \\ \operatorname{tg} i_2 &= \frac{\sin(\Omega_1 - \Omega_j)}{\sin \Omega_2 [\operatorname{ctg} i_1 \cos i_j + \sin i_j \cos(\Omega_1 - \Omega_j)]}, \\ \operatorname{tg}(\omega_1 - \omega_2) &= \frac{\sin(\Omega_1 - \Omega_j)}{\sin i_1 \operatorname{ctg} i_j - \cos i_1 \cos(\Omega_1 - \Omega_j)}. \end{aligned}\right\} \quad (2)$$

This provides the possibility of computing the values ω, i and $\bar{\Omega}$ searched for. The latter is obtained from the formula $\bar{\Omega} = \Omega_2 - u_j$ where u_j is the argument of Jupiter's latitude (which we assume to be moving along a circle).

#4. UTILIZED SYSTEMS OF OSCULATING ANGULAR ELEMENTS OF THE FIRST TEN MINOR PLANETS BROUGHT TO THE SAME FORM

The thus carried out computations gave the following elements:

Table 2

№ п/п	M	ω	Ω	i
		№ 1. Церера (CERES)		
1	291°.506	62°.761	+1406°.408	9°.389
2	290 .766	63 .420	1406 .339	9 .389
3	1 680 .327	64 .950	+ 866 .121	9 .399
4	4 121 .033	65 .324	− 81 .499	9 .380
5	4 433 .381	65 .723	202 .906	9 .381
6	4 433 .306	65 .737	202 .912	9 .378
7	4 433 .370	65 .727	202 .912	9 .378
8	9 073 .886	66 .149	2004 .923	9 .384
9	9 670 .340	69 .208	2237 .830	9 .369
10	10 056 .684	68 .918	2387 .270	9 .373
11	11 070 .032	69 .647	2780 .080	9 .371
12	11 156 .080	68 .511	2814 .347	9 .367
13	11 171 .511	68 .500	2820 .333	9 .368
14	11 830 .382	67 .239	−3075 .524	9 .368
		№ 2. Паллада - (PALLAS)		
1	41°.274	306°.961	+1464°.292	34°.256
2	100 .078	306 .696	1441 .451	34 .332
3	100 .446	306 .425	1441 .454	34 .281
4	133 .612	306 .416	1428 .487	34 .285
5	646 .250	306 .364	1228 .952	34 .260
6	647 .854	306 .436	1228 .980	34 .305
7	1 933 .391	306 .109	728 .685	34 .252
8	2 208 .096	306 .113	+ 621 .830	34 .252
9	5 307 .777	306 .747	− 584 .539	34 .357
10	8 711 .659	306 .788	1908 .366	34 .348
11	8 977 .583	307 .791	2011 .816	34 .348
12	9 110 .188	307 .426	2063 .932	34 .381
13	9 418 .704	307 .588	2184 .206	34 .388
14	9 619 .753	307 .473	2262 .320	34 .382
15	9 683 .821	307 .325	2287 .227	34 .380
16	10 222 .360	307 .060	2496 .661	34 .381
17	10 685 .220	307 .635	2677 .235	34 .447
18	10 788 .968	307 .927	2717 .698	34 .464
19	11 084 .962	308 .061	2832 .731	34 .469
20	11 461 .274	307 .793	−2978 .813	34 .461
		№ 3. Юнона - (Euno)		
1	349°.301	236°.552	+1382°.988	12°.740
2	762 .386	236 .310	1231 .232	12 .736
3	762 .739	236 .181	1231 .220	12 .739
4	844 .553	236 .408	1200 .967	12 .739
5	1 256 .978	236 .391	1049 .150	12 .733
6	1 504 .202	237 .748	957 .984	12 .734
7	1 616 .638	236 .692	916 .694	12 .728
8	2 151 .724	236 .546	+ 720 .143	12 .721
9	5 044 .416	237 .477	− 344 .444	12 .705
10	5 044 .416	237 .477	344 .447	12 .704
11	8 188 .600	239 .234	1501 .831	12 .671
12	9 317 .957	240 .041	1917 .762	12 .632
13	9 570 .988	239 .223	2010 .171	12 .648
14	10 066 .444	239 .923	2192 .884	12 .633
15	10 382 .951	239 .946	2309 .316	12 .644
16	10 808 .402	239 .949	2465 .943	12 .641
17	11 042 .130	240 .177	2552 .002	12 .649
18	11 357 .958	239 .976	2667 .999	12 .651
19	11 459 .895	239 .833	2705 .393	12 .652
20	11 582 .905	239 .723	2750 .601	12 .649
21	11 684 .041	239 .821	2787 .834	12 .646
22	12 198 .554	240 .663	−2977 .697	12 .613

Table 2 (contin'd)

№ п/п	M	ω	☊	i
		№ 4. Веста (Vesta)		
1	216°.080	145°.646	+1158°.718	5°.829
2	216 .716	145 .099	1158 .791	5 .828
3	811 .419	145 .453	976 .688	5 .829
4	1010 .113	145 .314	+ 915 .878	5 .828
5	4878 .338	146 .194	− 268 .137	5 .822
6	4878 .342	146 .189	268 .137	5 .822
7	4876 .339	147 .326	268 .857	5 .831
8	4878 .208	146 .222	268 .136	5 .861
9	6105 .231	145 .671	643 .510	5 .824
10	11627 .508	147 .309	−2334 .112	5 .831
		№ 5. Астрея (Astrea)		
1	318°.679	342°.446	+ 114°.424	4°.444
2	318 .864	342 .333	114 .395	4 .446
3	318 .751	342 .410	114 .401	4 .445
4	318 .751	342 .413	+ 114 .401	4 .445
5	666 .326	341 .688	− 7 .070	4 .448
6	666 .341	341 .681	7 .067	4 .448
7	1807 .014	342 .492	404 .957	4 .439
8	4904 .067	342 .070	1485 .630	4 .456
9	5017 .580	342 .830	−1524 .926	4 .451
		№ 6. Геба (Hebe)		
1	275°.225	233°.807	+ 57°.194	13°.815
2	275 .478	233 .817	+ 57 .193	13 .814
3	2498 .047	233 .209	− 650 .988	13 .794
4	5324 .339	233 .462	1551 .313	13 .812
5	8323 .973	233 .891	2506 .655	13 .811
6	9101 .095	234 .559	2754 .640	13 .764
7	9215 .829	234 .641	2791 .196	13 .761
8	9315 .352	234 .719	2834 .435	13 .762
9	9907 .138	234 .724	−3011 .447	13 .765
		№ 7. Ирида (Iris)		
1	330°.698	138°.159	+ 164°.245	6°.713
2	330 .941	137 .936	164 .200	6 .713
3	526 .128	137 .857	103 .560	6 .709
4	526 .119	137 .859	+ 103 .562	6 .710
5	5049 .089	137 .861	−1414 .855	6 .709
6	6685 .222	137 .553	1922 .205	6 .711
7	8148 .999	139 .411	2378 .256	6 .712
8	8340 .072	139 .529	2437 .712	6 .720
9	8437 .115	139 .826	2468 .127	6 .731
10	8538 .591	140 .058	2499 .701	6 .733
11	8637 .001	140 .054	2530 .278	6 .733
12	8807 .926	140 .003	2583 .466	6 .732
13	8904 .537	139 .998	2613 .464	6 .732
14	9000 .941	139 .894	2643 .318	6 .733
15	9067 .263	139 .880	−2673 .317	6 .733

Table 2 (end)

№ п/п	M	ω	Ω	i
		№ 8. Флора (Flora)		
1	35°.837	279°.514	+ 14°.186	4°.608 *
2	5765 .680	280 .684	− 1565 .397	4 .627
3	7613 .404	279 .537	2072 .304	4 .612
4	9590 .690	280 .663	− 2617 .[illegible]71	4 .624
		№ 9. Метида (Metis)		
1	144°.328	11°.033	− 49°.135	4°.515
2	145 .114	11 .038	49 .384	4 .515
3	145 .114	11 .037	49 .383	4 .514
4	1136 .944	11 .186	357 .855	4 .518
5	1137 .076	11 .040	357 .842	4 .503
6	1137 .218	11 .977	− 357 .894	4 .519
		№ 10. Хигиея - (Hygiea).		
1	481°.591	306°.002	+ 73°.162	5°.098
2	481 .532	305 .814	+ 73 .495	5 .092
3	3531 .338	310 .601	− 1362 .647	5 .117
4	4684 .331	310 .626	1902 .242	5 .117
5	4991 .678	306 .247	2045 .672	5 .104
6	4993 .536	306 .242	2045 .752	5 .112
7	5221 .962	306 .451	2152 .985	5 .112
8	5665 .099	306 .112	2360 .763	5 .109
9	6066 .697	306 .084	2550 .329	5 .102
10	6233 .586	307 .558	2628 .960	5 .105
11	6739 .084	311 .714	− 2869 .710	5 .117

* The average was taken from five values of the systems of osculating elements for 1848. I. 1.0.

Given are in the preceding Table: number of the system of elements taken from # 1; the mean anomaly M, computed by the above-described method for the osculation moment; the angular distance of the perihelion from the node of the plane of asteroid's orbit in the orbital plane of Jupiter (ω); the longitude of the node $\overline{\Omega}$, computed by the just-described method, and the inclination of the plane of asteroid's orbit to the orbital plane of Jupiter. (i).

#5. GENERAL REMARKS ON THE METHOD OF COMPUTATION OF INTERPOLATIVE ANOMALIES

According to N. D. Moiseyev [1], the quantities M, ω, and Ω are part of the expression for the interpolative anomaly μ as follows:

$$\mu = s_1 M + s_2 \omega + s_3 \Omega, \qquad (3)$$

where s_1, s_2 and s_3 are some constant coefficients.

The problem of determining or establishing the interpolative anomaly μ consists in so selecting the coefficients s_1, s_2 and s_3, that the interpolative anomaly μ, determined by the formula (3), remain "almost constant" for the given asteroid during the examined time interval, encompassed by the utilized systems of osculating elements. Such problems requires either statistical processing of the system of osculating angular elements of the asteroid. One of such possible methods is the standard method of least squares. Here we compose a conditional equation of the form (3), where the coefficients s_1, s_2 and s_3, just as are considered unknown. After that a corresponding system of normal equations is being composed, from which the most probable values of s_1, s_2, s_3 and μ are derived.

The correlation method for numelous variables may serve as the second method for the solution of the same problem. Here formula (3) is considered as a regression equation with unknown coefficients s_1, s_2 and s_3. The equivalence of both methods is obvious. Indeed, in both cases considered is the minimum of certain sum S, which in our case is equal to

../..

$$S=\sum_{g=1}^{n}(s_1\delta M_g+s_2\delta\omega_g+s_3\delta\overline{\Omega}_g)^2, \qquad (4)$$

where

$$\delta M_g=M_g-\bar{M},\quad \delta\omega_g=\omega_g-\bar{\omega},\quad \delta\overline{\Omega}_g=\overline{\Omega}_g-\overline{\overline{\Omega}}; \qquad (5)$$
$$(g=1,\ 2,\ldots,n).$$

$\overline{M}$, $\bar{\omega}$ and $\overline{\Omega}$ are the mean values of our quantities M_g , ω_g and $\overline{\Omega}_g$; g is the number of their values borrowed from the initial Table.

$$\left.\begin{matrix} t_1 & M_1 & \omega_1 & \overline{\Omega}_1 & i_1, \\ t_2 & M_2 & \omega_2 & \overline{\Omega}_2 & i_2, \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ t_n & M_n & \omega_n & \overline{\Omega}_n & i_n. \end{matrix}\right\} \qquad (6)$$

Table of # 4 is in our case such a table for each of the examined ten asteroids.

Rather than apply in the present work one of the two above-indicated methods of determination of the quantities s_1, s_2, s_3 and μ, we shall make use here of a certain combination of both methods, leading to the objective the quickest possible way: In order to demonstrate the existence of a general and partial correlative link between the quantities M, ω, Ω and to compute the coefficients s_1, s_2, s_3 themselves, we shall utilize the apparatus of the theory of linear correlation. As to the computation of errors in the coefficients s_1, s_2, s_3 and in the interpolative anomaly μ, both methods are used for that purpose — the method of the solution of normal equations, as well as the theory of linear correlation for many variables.

These methods are described in Romanovskiy's book [3] in the general case. We shall describe in the next paragraph the formulae and the method of computation applied in the current work.

#6. FORMULAE OF THE METHOD APPLIED FOR FINDING THE INTERPOLATIVE ANOMALY AND THE COEFFICIENTS S_1, S_2 AND S_3

In regard to coefficients s_1, s_2, s_3 and the quantity , it ought to be noted that they may be only obtained with a precision to a certain common numerical factor. Thus, for the purpose of definiteness, we shall alternately postulate the coefficients at M, ω, and $\overline{\Omega}$ as equal to the unity. This will liberate us from the just-indicated uncertainty.

For the sake of definiteness, let the coefficient at M by the unity. The cases of the coefficients equalling the unity at ω and $\overline{\Omega}$ are examined analogously.

Thus, we consider the interpolative anomaly of the form:

$$\mu_1 = M + s_2^{(1)}\omega + s_3^{(1)}\overline{\Omega}. \tag{7}$$

We already mentioned, that for finding s_1, s_2, s_3 we must find t the minimum of the sum:

$$S^{(1)} = \sum_{g=1}^{n} (\delta M_g + s_2^{(1)}\delta\omega_g + s_3^{(1)}\delta\overline{\Omega}_g)^2 \tag{8}$$

by a proper assortment of coefficients $s_2^{(1)}$ and $s_3^{(1)}$. In nonsingular cases the coefficients $s_2^{(1)}$ and $s_3^{(1)}$ are found from the equations:

$$\frac{\partial S^{(1)}}{\partial s_2^{(1)}} = 0; \quad \frac{\partial S^{(1)}}{\partial s_3^{(1)}} = 0. \tag{9}$$

Provided these equations are written in expanded form, we may substitute the quantities δM_g, $\delta\omega_g$ and $\delta\overline{\Omega}_g$ by the characteristics of their distributions, i.e. by means-square deflections σ_1 σ_2 and σ_3 respectively, and the coefficients of mutual correlation r_{12}, r_{21}, r_{23},

r_{13} and r_{31}. These distribution characteristics are found by the following formulae:

$$\left.\begin{aligned}
&\sigma_1^2=\frac{1}{n}\sum_{g=1}^{n}\delta M_g^2, \quad \sigma_2^2=\frac{1}{n}\sum_{g=1}^{n}\delta\omega_g^2, \quad \sigma_3^2=\frac{1}{n}\sum_{g=1}^{n}\delta\overline{\Omega}_g^2,\\
&r_{12}=r_{21}=\frac{\sum_{g=1}^{n}\delta M_g\,\delta\omega_g}{n\sigma_1\sigma_2}; \quad r_{23}=r_{32}=\frac{\sum_{g=1}^{n}\delta\omega_g\,\delta\overline{\Omega}_g}{n\sigma_2\sigma_3};\\
&r_{13}=r_{31}=\frac{\sum_{g=1}^{n}\delta\overline{\Omega}_g\,\delta M_g}{n\sigma_3\sigma_1}.
\end{aligned}\right\} \tag{10}$$

Then our equations (9) take the form:

$$\left.\begin{aligned}
\sigma_1\cdot r_{12}+s_2^{(1)}\cdot\sigma_2+s_3^{(1)}\sigma_3 r_{23}=0,\\
\sigma_1\cdot r_{13}+s_2^{(1)}\cdot\sigma_2 r_{23}+s_3^{(1)}\sigma_3=0.
\end{aligned}\right\} \tag{11}$$

The solution of euc ions (11), or, what is the same, of equations (10) or (9), is written in the form:

$$s_2^{(1)}=\frac{\sigma_1}{\sigma_2}\cdot\frac{D_{12}}{D_{11}}; \quad s_3^{(1)}=\frac{\sigma_1}{\sigma_3}\cdot\frac{D_{31}}{D_{11}}, \tag{12}$$

where D_{ik} is the substance of the cofactor:

$$D=\begin{vmatrix} 1 & r_{12} & r_{13}\\ r_{21} & 1 & r_{23}\\ r_{31} & r_{32} & 1 \end{vmatrix}. \tag{13}$$

Similarly resolved is the problem in the case when the coefficients in the interpolative anomalies are taken as unities at ω and $\overline{\Omega}$. The following formulae are then obtained:

$$\mu_2 = s_1^{(2)} M + \omega + s_3^{(2)} \overline{\Omega}, \tag{14}$$

$$\mu_3 = s_1^{(3)} M + s_2^{(3)} \omega + \overline{\Omega}, \tag{15}$$

where

$$s_1^{(2)} = \frac{\sigma_2}{\sigma_1} \cdot \frac{D_{12}}{D_{22}}; \quad s_3^{(2)} = \frac{\sigma_2}{\sigma_3} \cdot \frac{D_{23}}{D_{22}}, \tag{16}$$

$$s_1^{(3)} = \frac{\sigma_3}{\sigma_1} \cdot \frac{D_{31}}{D_{33}}; \quad s_2^{(3)} = \frac{\sigma_3}{\sigma_2} \cdot \frac{D_{23}}{D_{33}}. \tag{17}$$

As may be easily seen, the three regression equations we obtained are independent, and that is why there will be three interpolative anomalies and not one. These are determined by the formulae:

$$\mu_1 = \overline{M} + s_2^{(1)} \overline{\omega} + s_3^{(1)} \overline{\Omega}, \tag{18}$$

$$\mu_2 = s_1^{(2)} \overline{M} + \overline{\omega} + s_3^{(2)} \overline{\Omega}, \tag{19}$$

$$\mu_3 = s_1^{(3)} \overline{M} + s_2^{(3)} \overline{\omega} + \overline{\Omega}. \tag{20}$$

By their meaning itself, these quantities $s_2^{(1)}$ and $s_3^{(1)}$ are the substance of the regression coefficients of the quantities ω and $\overline{\Omega}$ for $-M$, thus:

$$-M = \mu_1 + s_2^{(1)} \omega + s_3^{(1)} \overline{\Omega} \tag{21}$$

or

$$-\delta M = s_2^{(1)} \delta\omega + s_3^{(1)} \delta\overline{\Omega}. \tag{22}$$

Similarly:

$$-\delta\omega = s_1^{(2)} \delta M + s_3^{(2)} \delta\overline{\Omega}, \tag{23}$$

$$-\delta\overline{\Omega} = s_1^{(3)} \delta M + s_2^{(3)} \delta\omega. \tag{24}$$

Using these formulae one may compute one of our elements by the two others. It is obvious that the so-calculated values of the elements will differ, generally speaking, from those given in the utilized table. The deflections of the computed values from the given ones will be characterized by the mean quadratic deflections Σ_1, Σ_2, and Σ_3 respectively for M, ω and $\overline{\Omega}$. It is well known from the theory of linear correlation for many variables that they may be found in the following manner:

$$\Sigma_1 = \sigma_1 \sqrt{\frac{D}{D_{11}}},\ \Sigma_2 = \sigma_2 \sqrt{\frac{D}{D_{22}}},\ \Sigma_3 = \sigma_3 \sqrt{\frac{D}{D_{33}}}, \qquad (25)$$

where D_{11}, D_{22}, D_{33} are cofactors of the elements of determinant's main diagonal (13).

After this it is sufficient to multiply these mean quadratic deflections by the quantity $\sqrt{\frac{n}{n-2}}$ in order to obtain the mean quadratic error per weight unit respectively in the conditional equations:

$$\sqrt{\frac{n}{n-2}}\ \Sigma_1 \text{ в } \delta M + s_2^{(1)}\delta\omega + s_3^{(1)}\delta\bar{\Omega} = 0,$$
$$\sqrt{\frac{n}{n-2}}\ \Sigma_2 \text{ в } s_1^{(2)}\delta M + \delta\omega + s_3^{(2)}\delta\bar{\Omega} = 0,$$
$$\sqrt{\frac{n}{n-2}}\ \Sigma_3 \text{ в } s_1^{(3)}\delta M + s_2^{(3)}\delta\omega + \delta\bar{\Omega} = 0,$$

provided the quantities $s_i^{(k)}$ $(k = 1, 2, 3;\ i = 1, 2, 3;\ i \neq k)$ are considered as those searched for, while δM, $\delta\omega$ and $\delta\bar{\Omega}$ are taken from the Table (6).

Having found the mean square error per unit of weight it is not difficult to compute the mean square errors of the coefficients $s_i^{(k)}$. To do this we still must know the weight of our unknowns. If we transform the formulae of the weights of our unknowns derived from the theory of normal equations, we shall obtain, by introducing in these formulae the distribution characteristics of the quantities M, ω, and $\bar{\Omega}$:

$$p_{s_i^{(k)}} = n\sigma_i^2 D_{kk}. \qquad (26)$$

Then, the mean square errors of the regression coefficients are:

$$\sigma_{s_i^{(k)}} = \sqrt{\frac{n}{n-2}} \frac{\Sigma_k}{\sqrt{p_{s_i^{(k)}}}} \qquad (27)$$
$$(i=1,\ 2,\ 3;\ k=1,\ 2,\ 3).$$

It is easy to see that the whole of the just-described method is valid not only for the case of three variables, but also for the case of any number of variables, for in the latter case all formulae from (3) through (27) are easily generalized.

#7. RESULTS OF COMPUTATIONS. CORRELATION COEFFICIENTS

Passing to the review of the concrete results of our computations, we shall begin with the correlation coefficients. The partial correlation coefficient between the quantities M and $\bar{\Omega}$ resulted greater for all the ten minor planets, than the two remaining two partial correlation coefficients. The former differs from the unity by two millionths (for Hygiea) in the worst case. This should not be the cause for surprise, for the quantities M and $\bar{\Omega}$ vary almost proportionally to the time. As to the remaining partial correlation coefficients between M and ω, ω and $\bar{\Omega}$, they are obtained notably different from the unity. The numerical values of the obtained correlation coefficients are given in page 23.

The fact that the particl correlation coefficient r_{12} was obtained positive, and r_{23} — negative, shows that the perihelia of all ten minor planets move counterclockwise if one looks from the northern pole of Jupiter's orbit. At the same time for three of the ten planets the correlation coefficients r_{12} and r_{23} resulted smaller than 0.7. This is so because the course of secular variation of ω is distorted by various short-periodical oscillations, and also by the errors in the very determinations of elements of the asteroids' orbit.

	Астероид Asteroid	r_{12}	r_{23}	r_{31}
1.	Ceres	+0.914415	−0.914470	−0.99999994
2.	Pallas	+0.830808	−0.830902	−0.99999992
3.	Euno	+0.972788	−0.972812	−0.99999987
4.	Vesta	+0.790614	−0.790837	−0.9999999
5.	Astrea	+0.287193	−0.287046	−0.9999999
6.	Hebe	+0.751090	−0.751126	−0.9999999
7.	Jris	+0.777383	−0.772215	−0.999[illegible]
8.	Flora	+0.559132	−0.559708	−0.9999[illegible]
9.	Metis	+0.783003	−0.783308	−0.9999[illegible]
10.	Hygiea	+0.494859	−0.496041	−0.99999855

8. RESULTS OF COMPUTATIONS OF INTERPOLATORY ANOMALIES

By the strength of the indicated causes, and also because of a comparatively small variation of ω, the regression coefficients at ω in the interpolative anomalies μ_1 and μ_2 are unreliably determined, just as are the regression coefficients M and Ω in the expression for μ_2. These expressions for the interpolative anomalies μ_1, μ_2, and μ_3 in the numerical form, are written as follows (the respective mean square errors in the regression coefficients are in parentheses; Σ_i $(i = 1,2,3)$ are the errors in the interpolative anomalies) :

(In the Table which follows the mean square errors are given at interpolative anomalies. As to the way to compute them, it will be examined in the next paragraph)

MEAN SQUARE ERRORS AT INTERPOLATIVE ANOMALIES

№ 1. Церера (CERES)

$3690 = M + 0.67\,(\pm\,0{,}25)\,\omega + 2.5725\,(\pm\,0.0003)\,\overline{\Omega};$ $\Sigma_1 = 0{,}75$

$14000 = 3.5\,(\pm\,1.0)\,M + \omega + 8.7\,(\pm\,3.0)\,\overline{\Omega};$ $\Sigma_2 = 1.7$

$1536 = 0.38830\,(\pm\,0.00005)\,M + 0.26\,(\pm\,0.09)\,\omega + \overline{\Omega};$ $\Sigma_3 = 0.29.$

№ 2. Паллада (PALLAS)

$4500 = M + 2.31\,(\pm\,0.73)\,\omega + 2.5707\,(\pm\,0.00025)\,\overline{\Omega};$ $\Sigma_1 = 1.1$

$600 = 0.077\,(\pm\,0.032)\,M + \omega + 0.198\,(\pm\,0.086)\,\overline{\Omega};$ $\Sigma_2 = 0.27$

$1750 = 0.38899\,(\pm\,0.00005)\,M + 0.89\,(\pm\,0.139)\,\omega + \overline{\Omega};$ $\Sigma_3 = 0.57$

№ 3. Юнона (EUNO)

$3820 = M + 1.22\,(\pm\,0.58)\,\omega + 2.7184\,(\pm\,0.00055)\,\overline{\Omega};$ $\Sigma_1 = 1.0$

$120 = 0.031\,(\pm\,0.015)\,M + \omega + 0.09\,(\pm\,0.04)\,\overline{\Omega};$ $\Sigma_2 = 0.16$

$1400 = 0.36785\,(\pm\,0.00007)\,M + 0.45\,(\pm\,0.20)\,\omega + \overline{\Omega};$ $\Sigma_3 = 0.34$

№ 4. Веста (VESTA)

$4400 = M + 2.7\,(\pm\,1.7)\,\omega + 3.2684\,(\pm\,0.0012)\,\overline{\Omega};$ $\Sigma_1 = 2.4$

$1100 = 0.24\,(\pm\,0.15)\,M + \omega + 0.79\,(\pm\,0.48)\,\overline{\Omega};$ $\Sigma_2 = 0.70$

$1350 = 0.30596\,(\pm\,0.00011)\,M + 0.83\,(\pm\,0.5)\,\omega + \overline{\Omega};$ $\Sigma_3 = 0.72$

№ 5. Астрея (ASTREA)

$370 = M - 0.8\,(\pm\,0{,}4)\,\omega + 2.8660\,(\pm\,0.00023)\,\overline{\Omega};$ $\Sigma_1 = 0.43$

$250 = -\,0.14\,(\pm\,0.07)\,M + \omega - 0.4\,(\pm\,0.21)\,\overline{\Omega};$ $\Sigma_2 = 0.18$

$130 = 0.34892\,(\pm\,0.0003)\,M - 0.29\,(\pm\,0.14)\,\omega + \overline{\Omega};$ $\Sigma_3 = 0.15$

№ 6. Геба (HEBE)

$600 = M + 0.6\,(\pm\,1.5)\,\omega + 3.1390\,(\pm\,0.0007)\,\overline{\Omega};$ $\Sigma_1 = 1.6$

$270 = 0.03\,(\pm\,0.07)\,M + \omega + 0.08\,(\pm\,0.22)\,\overline{\Omega};$ $\Sigma_2 = 0.35$

$190 = 0.31857\,(\pm\,0.00007)\,M + 0.19\,(\pm\,0.47)\,\omega + \overline{\Omega};$ $\Sigma_3 = 0.51$

№ 7. Ирида (IRIS)

$5100 = M - 43.3\,(\pm\,11.7)\,\omega + 3.083\,(\pm\,0.011)\,\overline{\Omega};$ $\Sigma_1 = 30.6$

$15 = -\,0.0110\,(\pm\,0.0043)\,M + \omega - 0.0333\,(\pm\,0.0042)\,\overline{\Omega};$ $\Sigma_2 = 0{,}223$

$1660 = 0.3275\,(\pm\,0.0036)\,M - 14.0\,(\pm\,3.9)\,\omega + \overline{\Omega};$ $\Sigma_3 = 10.0$

№ 8. Флора (Flora)

$1580 = M + 5.3\,(\pm\,11)\,\omega + 3.6324\,(\pm\,0.007)\,\overline{\Omega};$ $\Sigma_1 = 10.7$

$320 = 0.45\,(\pm\,0.3)\,M + \omega + 1.64\,(\pm\,1.0)\,\overline{\Omega};$ $\Sigma_2 = 3.1$

$436 = 0.2573\,(\pm\,0.0005)\,M + 1.46\,(\pm 3.0)\,\omega + \overline{\Omega};$ $\Sigma_3 = 2.8$

№ 9. Метида (METIS)

$200 = M + 19\,(\pm\,12)\,\omega + 3.2137\,(\pm\,0.0015)\,\overline{\Omega};$ $\Sigma_1 = 0.36$

$10 = 0.06\,(\pm\,0.04)\,M + \omega + 0.20\,(\pm\,0.12)\,\overline{\Omega};$ $\Sigma_2 = 0.02$

$62 = 0.31117\,(\pm\,0.00015)\,M + 6\,(\pm\,4)\,\omega + \overline{\Omega};$ $\Sigma_3 = 0.11$

№ 10. Хигиея (HYGIEA)

$1080 = M + 1.43\,(\pm\,0.8)\,\omega + 2.1291\,(\pm\,0.002)\,\overline{\Omega};$ $\Sigma_1 = 6.3$

$358 = 0.448\,(\pm\,0.14)\,M + \omega + 0.95\,(\pm\,0.3)\,\overline{\Omega};$ $\Sigma_2 = 1.9$

$506 = 0.46968\,(\pm\,0.00023)\,M + 0.67\,(\pm\,0.21)\,\omega + \overline{\Omega};$ $\Sigma_3 = 1.6$

#9. COMPUTATION OF DISPERSIONS OF INTERPOLATIVE ANOMALIES

The dispersion of the interpolative anomaly ^{2} is obviuosly equal to the quantity:

$$\sigma_{\mu}^{2}=\frac{1}{n}\sum_{g=1}^{n}[\mu-(s_1 M_g+s_2\omega_g+s_3\delta\overline{\Omega}_g)]^2. \qquad (28)$$

Introducing int~~o the~~ expression in place of M_g, ω_g and $\delta\overline{\Omega}_g$ their equivalents $\overline{M}+\delta M_g$, $\delta\omega_g$ and we shall have :

$$\sigma_{\mu}^{2}=\frac{1}{n}\sum_{g=1}^{n}(s_1\delta M_g+s_2\delta\omega_g+s_3\delta\overline{\Omega}_g)^2, \qquad (29)$$

or

$$\mu=s_1\overline{M}+s_2\bar{\omega}+s_3\delta\overline{\overline{\Omega}},$$

Thus, the dispersions of interpolative anomalies μ_1, μ_2 and μ_3 are equal respectively to the quantities Σ_1, Σ_2 and Σ_3 .

#10. DISCUSSION OF THE RESULTS OF COMPUTATION OF INTERPOLATIVE ANOMALIES

The above-presented material provides the possibility of reaching a series of conclusions.

Let us examine in the first place the question of the comparative quality of the obtained interpolative anomalies for each of the considered minor planets.

It is most natural to take as the criterion of this comparative quality the degree of satisfactory fulfillment of the "near-constancy" requirement for the interpolative anomaly within the examined time interval. As to its quantitative characteristic, one may take the degree of closeness to the unity of the general correlation coefficient. If we designate by R_1, R_2 and R_3 the respective general correlation coefficients for rhe interpolative anomalies μ_1, μ_2 and μ_3, we may derive formulae expressing them through partial correlation coefficients:

$$R_i = \sqrt{1 - \frac{D}{D_{ii}}}; \quad i = 1, 2, 3. \tag{30}$$

as this is being done in the theory of linear correlation for many variables (see for example the book by Romanovskiy [3]).

It is clear from these formulae, that the greater D_{ii}, the greater will be the corresponding total correlation coefficient R_i. But

$$D_{11} = 1 - r_{23}^2; \; D_{22} = 1 - r_{31}^2; \; D_{33} = 1 - r_{12}^2. \tag{31}$$

We thus may already estimate the quality of interpolative anomalies by the partial correlation coefficients themselves. Looking over the Table of these coefficients, given in #7, we see that the correlation coefficient r_{31} exceeds the remaining r_{12} and r_{23} for all the ten minor planets. We conclude on the basis of formulae (30) and (31) that the total correlation coefficient R_2 must be smaller than the remaining R_1 and R_3. This leads us to conclude that the first and the thirs interpolative anomalies are better for the first ten asteroids examined by us, than the second interpolative anomaly.

Further, the attention is called to the fact that the mean-square errors of the coefficients at ω exceed the corresponding mean-square errors for the coefficients at M and Ω.

Just as the preceding one, this fact is explained by the irregularity of the motion of the perihelion and by errors in the determination of the quantity ω.

The closeness of the coefficients at $\bar{\Omega}$ in the first anomaly, and near **M** in the third anomaly to the direct and inverse ratios of the mean motions of Jupiter and of the asteroid is also explained by the same fact. Besides, through more elaborate study of the results obtained one may see, that within the limits of coefficient errors, the latter are proportional to one another in the first and third interpolative anomalies, just as are the interpolatiwe anomalies themselves. This too is explained by the above-described cause.

In order to obtain a more complete representation on the quality of the interpolative anomalies constructed by us, it is interesting to compare them with those anomalies, which were utilized for minor planets under the denomination of "Delaunay anomalies".

#11. COMPARISON OF INTERPOLATIVE ANOMALIES WITH THE DELAUNAY ANOMALIES

We shall designate as the asculating Delaunay anomaly, or simple "Delaunay anomaly", the quantity :

$$\Delta = q_1 M + q_2 \bar{\Omega}, \qquad (32)$$

where the coefficients q_2 and q_1 are related among themselves as the mean motions of the asteroid and of Jupiter, while the mean motion of the former is taken for a specific osculation epoch. If we take for that osculation epoah the initial osculation epoch from Table 1, we shall obtain:

Ceres	$n : n_j$	2.572987
Euno	$n_j : n$	0.367661
Astrea	$n_j : n$	0.348462
Pallas	$n : n_j$	0.57277
Vesta	$n : n_j$	3.26914
Hebe	$n_j : n$	0.318559
Hygiea	$n_j : n$	0.469701

Because of the absence of a sufficient number of systems of osculating elements, such calculations have not been made for Flora and Metis. For Iris the mean motion for that epoch is determined very unreliably.

To compare the interpolative anomaly with the Delaunay anomaly, we shall examine the dispersions of both these anomalies. The dispersion of the latter is equal to that of the quantity:

$$\varepsilon_D = q_1 \delta M + q_2 \delta \overline{\Omega}, \tag{33}$$

or

$$\Delta = q_1 M + q_2 \overline{\Omega} \tag{34}$$

and

$$\sigma_D^2 = \frac{1}{n} \sum_{g=1}^{n} [\Delta - (q_1 M_g + q_2 \overline{\Omega}_g)]^2, \tag{35}$$

or

$$\sigma_D^2 = \frac{1}{n} \sum_{g=1}^{n} (q_1 \delta M_g + q_2 \delta \overline{\Omega})^2. \tag{36}$$

It is obvious that the dispersion of the interpolative anomaly $\mu = s_1 M + s_2 \omega + s_3 \Omega$ is equal to the dispersion of the quantity:

$$\varepsilon_\mu = s_1 \delta M + s_2 \delta\omega + s_3 \delta \overline{\Omega}, \tag{37}$$

as we already had the opportunity to see it in #9. The only thing that remains is the comparison of both quantities (33) and (37).

It is most appropriate to plot these values in a graph. All such graphs are presented at the end of the paper. It is obvious that only the first and the third interpolative anomlaies may be compared with the Delaunay anomaly. But for a better proportionality of the latter, providing a greater facility in graph construction for certain planets, the Delaunay and the interpolation anomalies were compared in the form:

$$\Delta_1 = M + \frac{n}{n_j}\Omega, \qquad (38)$$

$$\mu_1 = M + s_2^{(1)}\omega + s_3^{(1)}\bar{\Omega}. \qquad (7)$$

But for other minor planets it was done in the form:

$$\Delta^2 = \frac{n_j}{n}M + \bar{\Omega}, \qquad (39)$$

$$\mu_3 = s_1^{(3)}M + s_2^{(3)}\omega + \bar{\Omega}. \qquad (15)$$

These graphs clearly show that the interpolative anomalies are more constant quantities, than the Delaunay anomalies.

In conclusion, the author wishes to express his gratitude to his scientific guide — Professor N.D.Moiseyev, who provided him with valuable indications in the course of the completion of the present work, and who also contributed that valuable material on the systems of osculating elements for minor planets, from which the above-expounded results have sprung.

******* THE END *******

Translated by ANDRE L. BRICHANT

for the

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION.

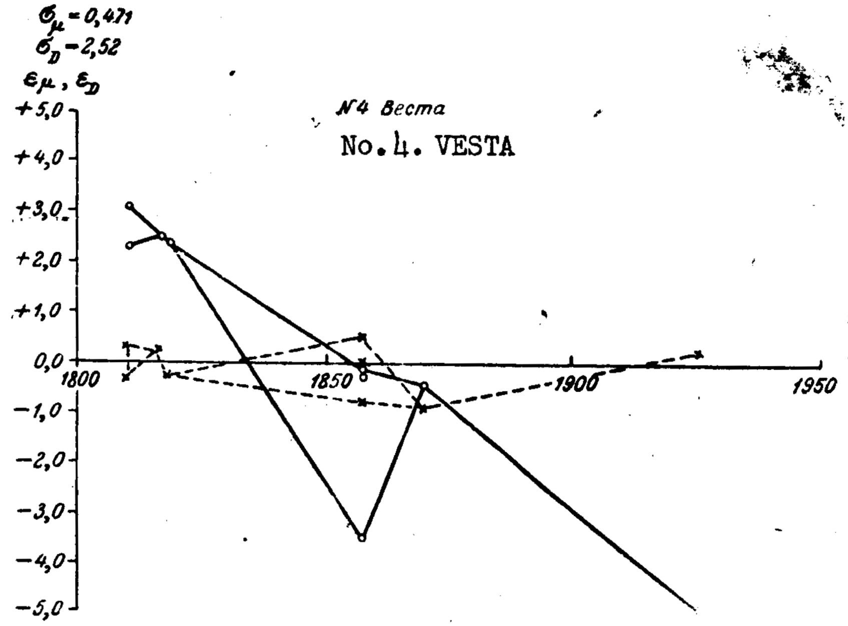


Рис. 1

Fig. 1.

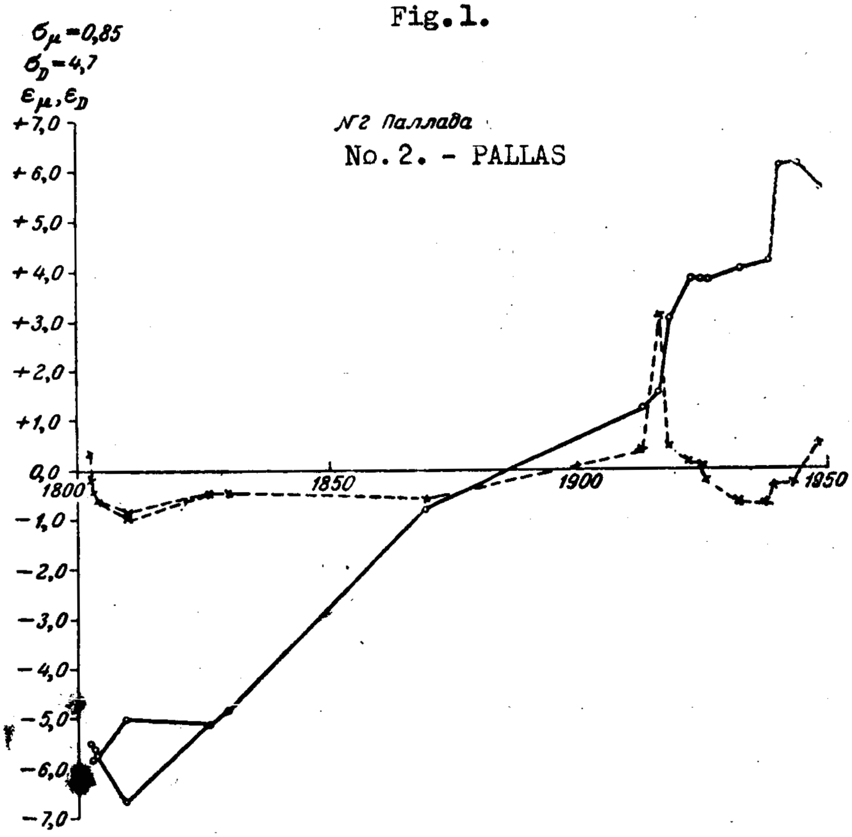


Fig. 2.

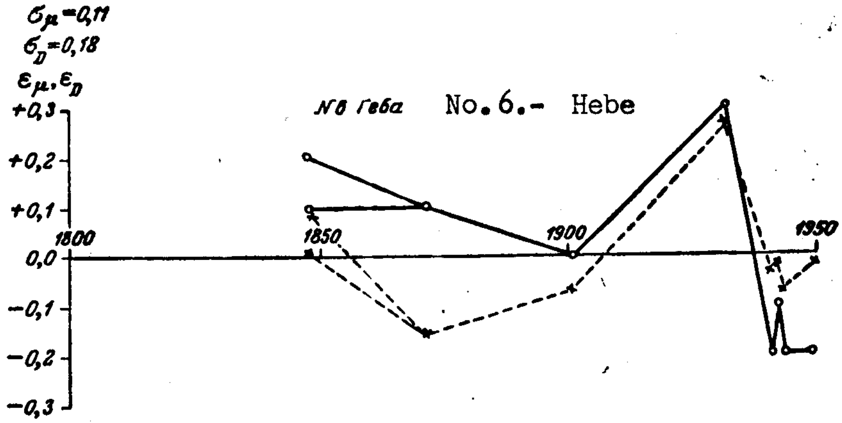
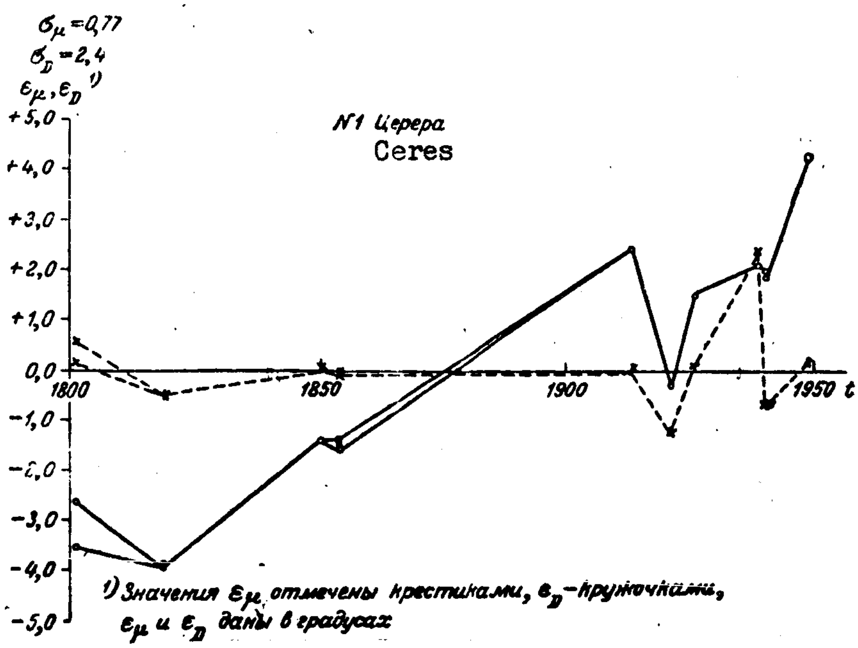


Рис. 3

Fig.3.



1) The values ε_μ are indicated by crosses, ε_D — by small circles and ε_μ and ε_D are given in degrees.

Fig.4.

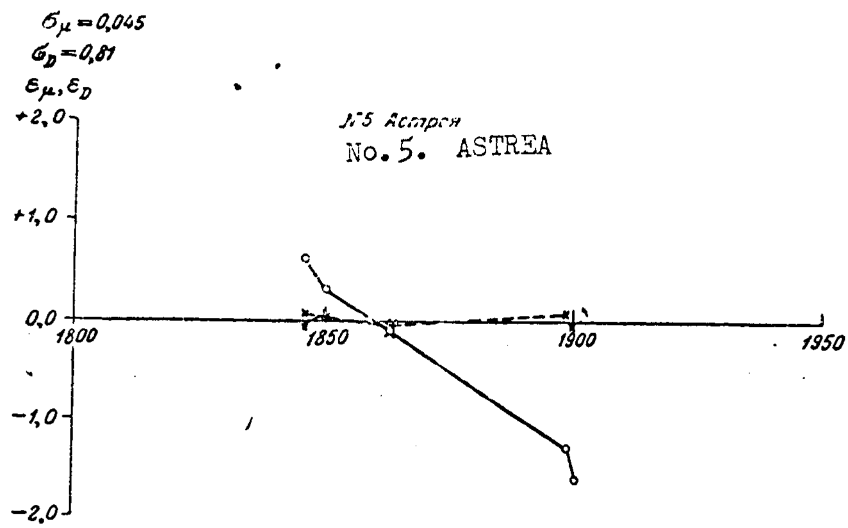


Рис. 5

Fig. 5

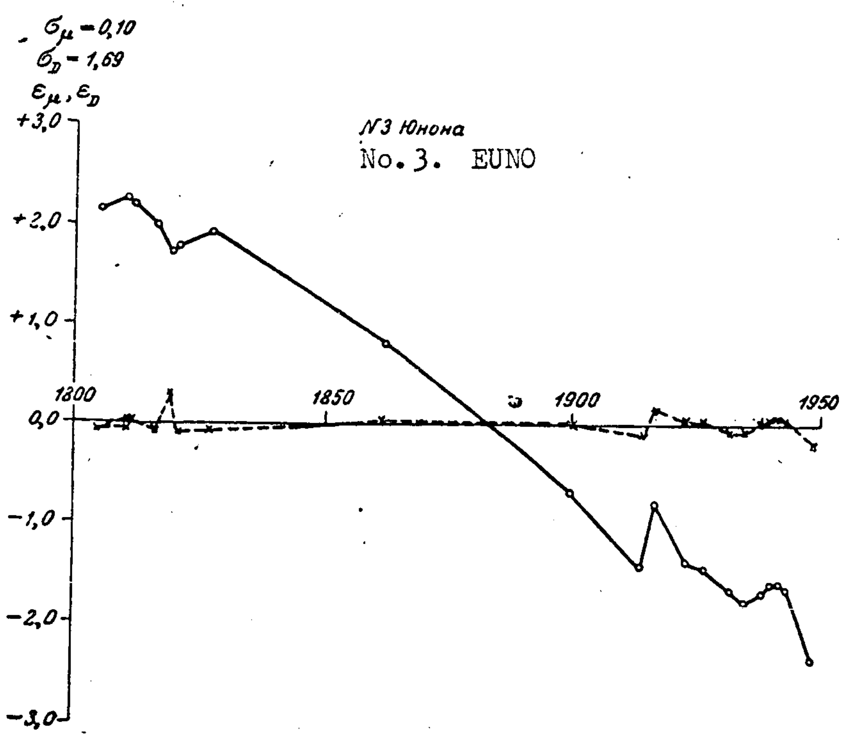


Fig. 6

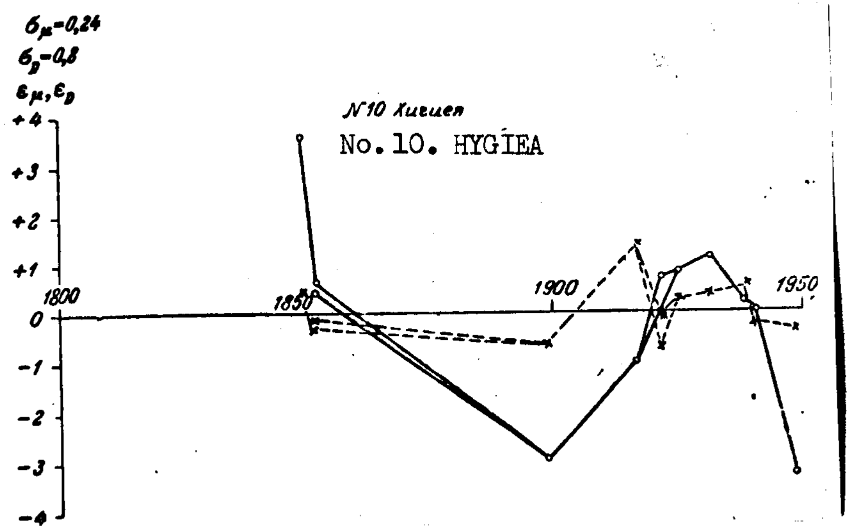


Fig. 7.

REFERENCES.

1. MOISEYEV, N. D. - Ob interpolyatsionno-osdrednennykh variantakh ogranichennoy zadachi trekh tochek. Vestnik MGU, No. 2, 1950.

2. ORLOV A. Ya. & ORLOV B. A. Kurs teoreticheskoy astronomii, M., L., GITTL, 1940.

3. ROMANOVSKIY, V. I. Natematicheskaya statistika, M.-L., GONTI, 1938

4. EPHEMERIDY MALYKH PLANET NA 1948, ITA A.N.SSSR, ch. 2., M. L., 1948.

5. Kleine Planeten. Bahnelemente und Oppositions — Ephemeriden. Bearbeit von dem Astr. Rechen Institut zu Berlin — Dahlem, Iahrgang 1917, 1925, 1932, 1933, 1934, 1935, 1936. 1937, 1938, 1939, 1940, 1941, 1942, 1943, 1944.

6. Leushner A. O. Reserach Surveys of the Orbit and Perturbations of Minor Planets 1 to 1091 from 1801. 0 to 1925. 5.